

Solving Quadratic Equations

Solving quadratic equations (equations with x^2 can be done in different ways. We will use three different methods in this packet. The first method is solving by factoring. For instance, if the equation was $x^2 - 22 = 9x$, you would have to subtract $9x$ from both sides of the equal sign so the equation would be $x^2 - 9x - 22 = 0$.

Solve by factoring: After the equation is set equal to 0, you factor the trinomial.

$$x^2 - 9x - 22 = 0$$

$$(x-11)(x+2) = 0$$

Now you would set each factor equal to zero and solve. Think about it, if the product of the two binomials equals zero, well then one of the factors has to be zero.

$$x^2 - 9x - 22 = 0$$

$$(x-11)(x+2) = 0$$

$$\begin{array}{r} x - 11 = 0 \quad x + 2 = 0 \\ \underline{+11 \quad +11} \quad \underline{-2 \quad -2} \end{array}$$

$$\begin{array}{l} x = 11 \quad \text{or} \quad x = -2 \\ x = \{-2, 11\} \end{array}$$

* Check in the ORIGINAL equation!

Solving each quadratic by factoring:

1) $x^2 - 5x - 14 = 0$

2) $x^2 + 11x = -30$

3) $x^2 - 45 = 4x$

Packet 6 Equations II

$$4) x^2 = 15x - 56$$

$$5) 3x^2 + 9x = 54$$

$$6) x^3 = x^2 + 12x$$

$$7) 25x^2 = 5x^3 + 30x$$

$$8) 108x = 12x^2 + 216$$

$$9) 3x^2 - 2x - 8 = 2x^2$$

Packet 6 Equations II

$$10) 10x^2 - 5x + 11 = 9x^2 + x + 83$$

$$11) 4x^2 + 3x - 12 = 6x^2 - 7x - 60$$

$$12) 6x^2 - 5x - 11 = 7x^2 - 5x - 47$$

$$13) 4x^2 - 9x + 3 = 2x^2 + 5x + 3$$

Q3 Quiz 9 Review

1) $x^2 = 22x - 96$

2) $5x^2 - 3x - 15 = 6x^2 + 10x + 15$

3) $8x^2 - 6x + 72 = 9x^2 - 6x - 72$

4) $5x = x^2 - 84$

Packet 6 Equations II

5) $6x^2 = 30x + 396$

6) $x^3 + 39x = 16x^2$

7) $20x^2 - 45 = 0$

8) $3x^2 = -3x + 216$

Packet 6 Equations II

$$9) 5x^3 - 210x = 5x^2$$

$$10) 5x^2 + 35x = 40$$

$$11) 21x^2 + 3x - 10 = 5x^2 + 3x - 9$$

$$12) x^2 = -22x - 72$$

Packet 6 Equations II

$$13) 11x^2 - 4x - 15 = 7x^2 - 4x - 14$$

$$14) 4x^2 - 17x + 3 = 6x^2 - 9x + 3$$

$$15) 8x^2 + 2x - 11 = 5x^2 + 9x - 11$$

$$16) 3x^2 - 6x + 8 = 12x^2 + 21x + 8$$

Answer Key:

$$1) x = \{6, 16\}$$

$$2) x = \{-10, -3\}$$

$$3) x = \{-12, 12\}$$

$$4) x = \{-7, 12\}$$

$$5) x = \{-6, 11\}$$

$$6) x = \{0, 3, 13\}$$

$$7) x = \{-\frac{3}{2}, \frac{3}{2}\}$$

$$8) x = \{-9, 8\}$$

$$9) x = \{-6, 0, 7\}$$

$$10) x = \{-8, 1\}$$

$$11) x = \{-\frac{1}{4}, \frac{1}{4}\}$$

$$12) x = \{-18, -4\}$$

$$13) x = \{-\frac{1}{2}, \frac{1}{2}\}$$

$$14) x = \{-2, 0\}$$

$$15) x = \{0, \frac{7}{3}\}$$

$$16) x = \{-3, 0\}$$

Solve using the quadratic formula:

Solve $x^2 - 9x - 22 = 0$ using the *quadratic formula*

When $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a is the coefficient of x^2 **b** is the coefficient of x **c** is the number (third term)

Notice the \pm is what will give your two answers (just like you had when solving by factoring)

$$x^2 - 9x - 22 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -9$$

$$c = -22$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-22)}}{2(1)} \rightarrow (-9)^2 - 4(1)(-22) \text{ can be done in one step in the calculator (leave out radical!!).}$$

$$x = \frac{9 \pm \sqrt{169}}{2}$$

Split and do the + side and - side

$$\frac{9 - 13}{2} \qquad \frac{9 + 13}{2}$$

$$x = -2 \qquad \text{or} \qquad x = 11$$

$$x = \{-2, 11\}$$

* *Check in the ORIGINAL equation!*

Remember, ALL SUBSTITUTIONS must be done in PARENTHESES!!!!!!!!

Packet 6 Equations II

Solving each quadratic using the Quadratic Formula:

14) $2x^2 - 6x + 1 = 0$

15) $3x^2 + 2x = 3$

16) $4x^2 + 2 = -7x$

17) $7x^2 = 3x + 2$

Packet 6 Equations II

$$18) 3x^2 + 6 = 5x$$

$$19) 9x - 3 = 4x^2$$

$$20) 4x^2 - 5x = 2$$

$$21) 11x^2 - 3 = -4x$$

Answer Key:

$$14) x = \frac{3}{2} \pm \frac{\sqrt{7}}{2}$$

$$15) x = \frac{-1}{3} \pm \frac{\sqrt{10}}{3}$$

$$16) x = \frac{-7}{8} \pm \frac{\sqrt{17}}{8}$$

$$17) x = \frac{3}{14} \pm \frac{\sqrt{65}}{14}$$

18) No Real Solution

$$19) x = \frac{9}{8} \pm \frac{\sqrt{33}}{8}$$

$$20) x = \frac{5}{8} \pm \frac{\sqrt{57}}{8}$$

$$21) x = \frac{-2}{11} \pm \frac{\sqrt{37}}{11}$$

Q__ Quiz __ Review

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You must write the quadratic formula for each problem

1) $5x^2 - 3 = -4x$

2) $6x^2 = 12x - 5$

Packet 6 Equations II

3) $2x^2 - 5x = 4$

4) $x^2 = -14x - 19$

Packet 6 Equations II

5) $4x^2 = 6x + 3$

6) $7x^2 + x + 3 = 0$

Packet 6 Equations II

7) $16x^2 + 1 = 12x$

8) $6x^2 = 7x + 1$

Packet 6 Equations II

$$9) 12x^2 - 13x - 4 = 0$$

$$10) 8x^2 + 16x + 3 = 0$$

Answer Key:

$$1) x = \frac{-2}{5} \pm \frac{\sqrt{19}}{5} \quad 2) x = 1 \pm \frac{\sqrt{6}}{6} \quad 3) x = \frac{5}{4} \pm \frac{\sqrt{57}}{4} \quad 4) x = -7 \pm \sqrt{30}$$

$$5) x = \frac{3}{4} \pm \frac{\sqrt{21}}{4} \quad 6) \text{No Real Solution } (\sqrt{-83}) \quad 7) x = \frac{3}{8} \pm \frac{\sqrt{5}}{8}$$

$$8) x = \frac{7}{12} \pm \frac{\sqrt{73}}{12} \quad 9) x = \{-\frac{1}{4}, \frac{4}{3}\} \quad 10) x = -1 \pm \frac{\sqrt{10}}{4}$$

Factor:

1) $x^2 + 4x + 4$

2) $x^2 - 6x + 9$

3) $x^2 - 18x + 81$

4) $x^2 + 10x + 25$

5) $x^2 - 20x + 100$

6) $x^2 + 8x + 16$

7) $x^2 - 22x + 121$

8) $x^2 + 32x + 256$

9) $x^2 - 40x + 400$

Completing the Square

Completing the square is another method that is used to solve quadratic equations. This method is especially helpful when the quadratic equation cannot be solved by simply factoring.

Remember the standard form for a quadratic equation is: $ax^2 + bx + c = 0$.

Example:

1. $x^2 + 8x - 9 = 0$

$$x^2 + 8x - 9 = 0$$

$$\quad \quad +9 \quad +9$$

$$x^2 + 8x = 9$$

$$\left(\frac{1}{2}(8)\right)^2 = (4)^2 = 16$$

$$x^2 + 8x + 16 = 9 + 16$$

$$x^2 + 8x + 16 = 25$$

$$(x + 4)(x + 4) = 25$$

$$(x + 4)^2 = 25$$

$$\sqrt{(x + 4)^2} = \sqrt{25}$$

$$x + 4 = \pm 5$$

$$x + 4 = 5 \qquad x + 4 = -5$$

$$\quad -4 \quad -4 \qquad \quad -4 \quad -4$$

$$x = 1 \qquad \qquad \qquad x = -9$$

$$x = \{-9, 1\}$$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.
4. To create the perfect square trinomial:
 - a) Take $\left(\frac{1}{2}b\right)^2$
 - b) Add that value to both sides of the equation.
5. Factor the perfect square trinomial.
6. Rewrite the factors as a squared binomial.
7. Take the square root of both sides.
8. Split the solution into two equations
9. Solve for x.
10. Create your final answer.

Packet 6 Equations II

More Examples:

$$1) x^2 + 2x - 3 = 0$$

$$2) x^2 - 16x + 60 = 0$$

$$3) x^2 - 8x + 7 = 0$$

Example:

$$1. \quad x^2 - 10x - 7 = 0$$

$$x^2 - 10x - 9 = 0$$
$$\quad \quad \quad +7 \quad +7$$

$$x^2 - 10x = 7$$

$$\left(\frac{1}{2}(-10)\right)^2 = (-5)^2 = 25$$

$$x^2 + 10x + 25 = 7 + 25$$

$$x^2 + 10x + 25 = 32$$

$$(x - 5)(x - 5) = 32$$

$$(x - 5)^2 = 32$$

$$\sqrt{(x - 5)^2} = \sqrt{32}$$
$$x - 5 = \pm 4\sqrt{2}$$
$$\quad \quad \quad +5 \quad +5$$

$$x = 5 \pm 4\sqrt{2}$$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.
4. To create the perfect square trinomial:
 - a) Take $\left(\frac{1}{2}b\right)^2$
 - b) Add that value to both sides of the equation.
5. Factor the perfect square trinomial.
6. Rewrite the factors as a squared binomial.
7. Take the square root of both sides.
- 8. Isolate X. Since you cannot combine it with $\pm 4\sqrt{2}$, you do not need to split it into two equations.**
9. Create your final answer

Packet 6 Equations II

4) $x^2 + 12x + 4 = 0$

5) $x^2 - 8x - 11 = 0$

6) $x^2 + 6x - 36 = 0$

7) $x^2 + 4x - 44 = 0$

8) $x^2 + 24x + 24 = 0$

9) $x^2 - 16x - 6 = 0$

Packet 6 Equations II

$$10) x^2 + 6x + 4 = 0$$

$$11) x^2 - 10x + 5 = 0$$

$$12) x^2 + 2x - 47 = 0$$

$$13) x^2 + 4x - 92 = 0$$

$$14) x^2 - 28x + 4 = 0$$

$$15) x^2 - 12x - 20 = 0$$

Packet 6 Equations II

16) $x^2 + 18x + 1 = 0$

17) $x^2 - 22x - 39 = 0$

18) $x^2 + 8x - 164 = 0$

Q___ Quiz ___ Review

Solve each quadratic using completing the square:

1) $x^2 + 6x - 112 = 0$

2) $x^2 - 20x + 64 = 0$

3) $x^2 - 10x + 7 = 0$

4) $x^2 + 8x + 8 = 0$

Packet 6 Equations II

5) $x^2 - 2x - 23 = 0$

6) $x^2 + 6x - 63 = 0$

7) $x^2 + 20x - 8 = 0$

8) $x^2 - 24x + 16 = 0$

Packet 6 Equations II

9) $x^2 - 18x - 88 = 0$

10) $x^2 + 12x - 39 = 0$

11) $x^2 + 4x - 60 = 0$

12) $x^2 - 14x + 17 = 0$

Packet 6 Equations II

13) $x^2 - 10x + 7 = 0$

14) $x^2 + 8x + 15 = 0$

15) $x^2 - 2x - 80 = 0$

16) $x^2 + 6x - 39 = 0$

Packet 6 Equations II

17) $x^2 + 20x - 25 = 0$

18) $x^2 - 24x + 23 = 0$

19) $x^2 - 18x - 40 = 0$

20) $x^2 + 12x - 18 = 0$

Packet 6 Equations II

$$21) x^2 + 15x + 26 = 0$$

$$22) x^2 - 10x - 25 = 0$$

Answer Key:

$$1) x = \{-14, 8\}$$

$$2) x = \{4, 16\}$$

$$3) x = 5 \pm 4\sqrt{2}$$

$$4) x = -4 \pm 2\sqrt{2}$$

$$5) x = 1 \pm 2\sqrt{6}$$

$$6) x = -3 \pm 6\sqrt{2}$$

$$7) x = -10 \pm 6\sqrt{3}$$

$$8) x = 12 \pm 8\sqrt{2}$$

$$9) x = \{-4, 22\}$$

$$10) x = -6 \pm 5\sqrt{3}$$

$$11) x = \{-10, 6\}$$

$$12) x = 7 \pm 4\sqrt{2}$$

$$13) x = 5 \pm 3\sqrt{2}$$

$$14) x = \{-5, -3\}$$

$$15) x = \{-8, 10\}$$

$$16) x = -3 \pm 4\sqrt{3}$$

Radical Equations

If you have the variable (usually x) under the radical, we cannot solve for it until we get it out from under the radical. The way to do this is to get the expression with the radical by itself and then square both sides. When the radical is by itself, squaring it gets rid of the radical. At this level, we will wind up with either a linear equation or a quadratic equation (both of which we covered in this packet).

Example 1: $7\sqrt{11x+3} - 46 = -4$

$$\begin{array}{r} 7\sqrt{11x+3} - 46 = -4 \\ \quad \quad \quad +46 \quad \quad +46 \\ \hline \end{array}$$

$$\frac{7\sqrt{11x+3}}{7} = \frac{42}{7}$$

$$\begin{array}{l} \sqrt{11x+3} = 6 \quad \text{*Radical is isolated, square both sides.. this gets rid of the radical on the left side of the equation.} \\ (\sqrt{11x+3})^2 = (6)^2 \end{array}$$

$$\begin{array}{r} 11x + 3 = 36 \\ \quad \quad \quad -3 \quad \quad \quad -3 \\ \hline \end{array}$$

$$\frac{11x}{11} = \frac{33}{11}$$

$$x = 3 \quad \text{*You MUST do your check to make sure your solution is not an extraneous root.}$$

Check: $7\sqrt{11x+3} - 46 = -4 \quad x = 3$

$$\begin{array}{r} 7\sqrt{11(3)+3} - 46 = -4 \\ 7\sqrt{33+3} - 46 = -4 \\ 7\sqrt{36} - 46 = -4 \\ 7(6) - 46 = -4 \\ 42 - 46 = -4 \\ -4 = -4 \quad \checkmark \end{array}$$

$$x = \{3\}$$

Packet 6 Equations II

Example 2:

$$\sqrt{13x + 30} - x = 0$$

$$\sqrt{13x + 30} \stackrel{+x}{=} x$$

$$(\sqrt{13x + 30})^2 = (x)^2 \quad (\text{Radical is isolated, square both sides})$$

$$\begin{array}{r} 13x + 30 = x^2 \\ \hline -13x - 30 \quad -13x - 30 \end{array}$$

$$0 = x^2 - 13x - 30 \quad (\text{Factor and solve})$$

$$0 = (x + 2)(x - 15)$$

$$x + 2 = 0 \quad x - 15 = 0$$

$$\begin{array}{r} -2 \quad -2 \quad +15 \quad +15 \\ \hline x = -2 \quad x = 15 \end{array} \quad (\text{Check for extraneous roots})$$

Check:

$$\sqrt{13x + 30} - x = 0$$

$$x = -2$$

$$\begin{array}{r} \sqrt{13(-2) + 30} - (-2) = 0 \\ \sqrt{-26 + 30} - (-2) = 0 \\ \sqrt{4} - (-2) = 0 \\ 2 - (-2) = 0 \\ 4 \neq 0 \end{array}$$

$$x = 15$$

$$\begin{array}{r} \sqrt{13(15) + 30} - (15) = 0 \\ \sqrt{195 + 30} - (15) = 0 \\ \sqrt{225} - (15) = 0 \\ 15 - (15) = 0 \\ 0 = 0 \quad \checkmark \end{array}$$

$$x = \{15\}$$

Packet 6 Equations II

Example 3: $\sqrt{12x - 20} - x = 1$
 $\quad\quad\quad +x \quad +x$

$$\sqrt{12x - 20} = x + 1 \quad (\text{Radical is isolated, square both sides})$$

$$\begin{array}{rcl} (\sqrt{12x - 20})^2 & = & (x + 1)^2 \\ (\sqrt{12x - 20})^2 & = & (x + 1)(x + 1) \\ 12x - 20 & = & x^2 + 1x + 1x + 1 \\ 12x - 20 & = & x^2 + 2x + 1 \\ -12x + 20 & & -12x + 20 \end{array}$$

$$0 = x^2 - 10x + 21 \quad (\text{Factor and solve})$$

$$\begin{array}{rcl} 0 & = & (x - 3)(x - 7) \\ & & x - 3 = 0 \quad x - 7 = 0 \\ & & +3 \quad +3 \quad +7 \quad +7 \\ & & x = 3 \quad x = 7 \quad (\text{Check for extraneous roots}) \end{array}$$

Check:

$$\sqrt{12x - 20} - x = 1$$

$$X = 3$$

$$\begin{array}{rcl} \sqrt{12(3) - 20} - (3) & = & 1 \\ \sqrt{36 - 20} - (3) & = & 1 \\ \sqrt{16} - (3) & = & 1 \\ 4 - (3) & = & 1 \\ 1 & = & 1 \quad \checkmark \end{array}$$

$$x = 7$$

$$\begin{array}{rcl} \sqrt{12(7) - 20} - (7) & = & 1 \\ \sqrt{84 - 20} - (7) & = & 1 \\ \sqrt{64} - (7) & = & 1 \\ 8 - (7) & = & 1 \\ 1 & = & 1 \quad \checkmark \end{array}$$

$$x = \{3, 7\}$$

Packet 6 Equations II

Example 4:
$$\sqrt{11x + 36} + 6 = x$$

$$\underline{-6 \qquad \qquad -6}$$

$$\sqrt{11x + 36} = x - 6 \quad (\text{Radical is isolated, square both sides})$$

$$\begin{aligned} (\sqrt{11x + 36})^2 &= (x - 6)^2 \\ (\sqrt{11x + 36})^2 &= (x - 6)(x - 6) \\ 11x + 36 &= x^2 - 6x - 6x + 36 \\ 11x + 36 &= x^2 - 12x + 36 \\ \underline{-11x \quad -36} &\qquad \underline{-11x \quad -36} \end{aligned}$$

$$0 = x^2 - 23x \quad (\text{Factor and solve})$$

$$\begin{aligned} 0 &= x(x - 23) \\ x = 0 \quad x - 23 &= 0 \\ &\quad \underline{+23 \quad +23} \\ &\quad x = 23 \end{aligned}$$

Check:

$$\sqrt{11x + 36} + 6 = x$$

$$x = 0$$

$$x = 7$$

$$\begin{aligned} \sqrt{11(0) + 36} + 6 &= (0) \\ \sqrt{0 + 36} + 6 &= 0 \\ \sqrt{36} + 6 &= 0 \\ 6 + 6 &= 0 \\ 12 &\neq 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \sqrt{11(23) + 36} + 6 &= (23) \\ \sqrt{253 + 36} + 6 &= 23 \\ \sqrt{289} + 6 &= 23 \\ 17 + 6 &= 23 \\ 23 &= 23 \quad \checkmark \end{aligned}$$

$$x = \{23\}$$

1) $\sqrt{14x + 1} = 13$

Packet 6 Equations II

$$2) 3\sqrt{8x + 9} - 24 = -3$$

$$3) \sqrt[3]{21 - 4x} + 17 = 20$$

Packet 6 Equations II

$$4) \sqrt{72 - x} = x$$

$$5) \sqrt{-19x - 60} = x$$

Packet 6 Equations II

$$6) -\sqrt{-19x - 60} = x$$

$$7) \sqrt{16x - 48} = x$$

Packet 6 Equations II

8) $\sqrt{12x + 4} - 3 = x$

9) $\sqrt{11x + 26} - x = 4$

Packet 6 Equations II

10) $\sqrt{12x + 4} - 2 = x$

11) $\sqrt{8x + 1} - x = -1$

Packet 6 Equations II

12) $\sqrt{41 - 2x} - 11 = x$

13) $\sqrt{8x + 25} - x = -5$

Packet 6 Equations II

$$14) \sqrt{-28 - 11x} = x$$

$$15) \sqrt{13x + 61} - 7 = x$$

Q__ Quiz __ Review:

$$1) \sqrt{7x - 6} = x$$

$$x = \{1, 6\}$$

$$2) 6\sqrt{6x - 2} - 55 = -7$$

$$x = 11$$

Packet 6 Equations II

$$3) \frac{1}{2} \sqrt{11x + 31} + 9 = 16$$

$$x = 15$$

$$4) \sqrt{x^2 - 48} + 2 = x$$

$$x = 13$$

Packet 6 Equations II

$$5) \sqrt{5x + 4} - x = -2$$

$x = \{9\}$, 0 is an extraneous root

$$6) \sqrt{14x - 6} - x = 3$$

$x = \{3, 5\}$

Packet 6 Equations II

$$7) \sqrt{10x - 1} + 4 = x$$

$x = \{17\}$, 1 is an extraneous root

$$8) \sqrt{9x + 1} - x = -1$$

$x = \{11\}$, 0 is an extraneous root

Packet 6 Equations II

$$9) \sqrt[3]{4} \sqrt{13x - 38} - 8 = 1$$

$$x = 14$$

$$10) \sqrt{11x + 49} - 7 = x$$

$$x = \{-3, 0\}$$